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32. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics in the Ohio University, Athens, Ohio.

If a conic be inscribed in a triangle and its focus move along a given straight line, the locus of the other focus is a conic circumscribing the triangle.

Solutions to these problems should be received on or before July 1st.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

9. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

The solids bounded by the surfaces whose equations are $\left[\frac{x}{a}\right]^{\frac{2}{3}}+\left[\frac{y}{b}\right]^{\frac{2}{3}}+\left[\frac{z}{c}\right]^{\frac{2}{3}}=1$ and $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ where $a>b>c$ have their centers coincident.

Find (1 and 2) the volume of each without the other, and (3) the volume common to both by direct integration, using the formula $V=\int\int\int dxdydz$.

Solution by the Proposer.

(1). From the equations we find for z,

$z=(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}$ and $z=\frac{c}{ab}(a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{2}{3}}y^{\frac{2}{3}}-b^{\frac{2}{3}}x^{\frac{2}{3}})^{\frac{3}{2}}$ eliminating z we find for the x—limits

$x=\frac{a}{b}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})(b^{\frac{2}{3}}-y^{\frac{2}{3}})}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{3}{2}}=x_1$ to $x=0$, the y—limits are $y=b$ to $y=0$.

The volume of that part of $x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ without $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}\left(\frac{z}{c}\right)^{\frac{2}{3}}=1$ we get $V=8\int_0^b\int_0^{x_1}(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}dydx-\frac{8c}{ab}\int_0^b\int_0^{x_1}(a^{\frac{2}{3}}b^{\frac{2}{3}}-a^{\frac{2}{3}}y^{\frac{2}{3}}-b^{\frac{2}{3}}x^{\frac{2}{3}})^{\frac{3}{2}}\,dydx$

$$V=8\int_0^b \Big/ \Big[\tfrac{1}{2}x(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}+\frac{3}{8}\,x(b^{\frac{2}{3}}-y^{\frac{2}{3}})(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{1}{2}}$$

$$-\frac{3}{16}x^{\frac{1}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{2}\left(b^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{\frac{1}{2}}+\frac{3}{16}$$

$$\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}\sin^{-1}\left\{\frac{x}{(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}}\right\}^{\frac{1}{3}}\Bigg]_{o}^{x_1}\Bigg/\,dy$$

$$-\frac{8c}{ab}\int_{o}^{b}\Bigg/\left[\frac{1}{2}x\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{3}{2}}+\frac{3}{8}xa^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)\right.$$

$$\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{1}{2}}$$

$$-\frac{3}{16}\frac{x^{\frac{1}{3}}a^{\frac{4}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{2}}{b^{\frac{2}{3}}}\left\{a^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{1}{2}}$$

$$\left.+\frac{3a^{2}}{16b}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}\sin^{-1}\left\{\frac{bx}{a(b^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}}\right\}^{\frac{1}{3}}\right]_{o}^{x_1}\Bigg/\,dy$$

$$V=\left[\frac{3}{2}\sin^{-1}\left\{\frac{a}{b}\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}-\frac{3ac}{2b^{2}}\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{3}{3}}}\right\}^{\frac{1}{3}}\right.$$

$$+\frac{3ac}{b^{2}}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^{2}}\right\}$$

$$\left.+\frac{3ac}{2b^{2}}\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}\right]\int_{o}^{b}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{3}dy.$$

$$V=\frac{8}{35}b^{3}\sin^{-1}\left\{\frac{a}{b}\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}-\frac{8}{35}abc\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}$$

$$+\frac{16}{35}abc\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^{2}}\right\}$$

$$+\frac{8}{35}abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}$$

(2) From the equations $x=\frac{a}{bc}\left\{c^{\frac{2}{3}}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)--b^{\frac{2}{3}}z^{\frac{2}{3}}\right\}^{\frac{3}{2}}$, and $x=\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-z^{\frac{2}{3}}\right)^{\frac{3}{2}}$

$\therefore$ z-limits are $z=\frac{c}{b}\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})(b^{\frac{2}{3}}-y^{\frac{2}{3}})}{\cdot c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}^{\frac{3}{2}}$ to $z=0$. y-limits$=b$ and 0.

Hence the volume of that part of $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}+\left(\frac{z}{c}\right)^{\frac{2}{3}}=1$, without

$x^{\frac{2}{3}}+y^{\frac{2}{3}}+z^{\frac{2}{3}}=b^{\frac{2}{3}}$ we get

$$V=8\frac{a}{bc}\int_0^b\int_0^{z_1}\left\{c^{\frac{2}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})-b^{\frac{2}{3}}z^{\frac{2}{3}}\right\}^{\frac{3}{2}}dydz-8\int_0^b\int_0^{z_1}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-z^{\frac{2}{3}}\right)^{\frac{3}{2}}dydz.$$

By writing a for c and c for a and $+$ for $-$ this integral is the same as the first, hence by symmetry.

$$V=\frac{8}{35}\,abc\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-a^{\frac{2}{3}}}{c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{8}{35}\,b^3\sin^{-1}\left\{\frac{c\,(b^{\frac{2}{3}}-a^{\frac{2}{3}})}{b\,(c^{\frac{2}{3}}-a^{\frac{2}{3}})}\right\}^{\frac{1}{3}}$$

$$-\frac{16}{35}\,abc\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})^{\frac{5}{2}}(c^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}(c^{\frac{2}{3}}-a^{\frac{2}{3}})^2}\right\}$$

$$-\frac{8}{35}\,abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}\left\{\frac{(b^{\frac{2}{3}}-a^{\frac{2}{3}})^{\frac{1}{2}}(c^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}-a^{\frac{2}{3}}}\right\}.$$

(3). For volume common to both x-limits are $x=x_1$ to $x=0$ and $x=\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^{\frac{3}{2}}=x_2$ to $x=x_1$, y-limits are $y=b$ to $y=0$.

$$\therefore\ V=8\int_0^b\int_0^{x_1}\frac{c}{ab}\left\{a^{\frac{2}{3}}(b^{\frac{2}{3}}-y^{\frac{2}{3}})-b^{\frac{2}{3}}x^{\frac{2}{3}}\right\}^{\frac{3}{2}}dydx$$

$$+8\int_0^b\int_{x_1}^{x_2}\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dydx$$

$$V=\Bigg[\frac{3}{4}\pi+\frac{3}{2}\frac{ac}{b^2}\sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{3}{2}\sin^{-1}\left\{\frac{a\,(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{b\,(a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}$$

$$-\frac{3ac}{b^2}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{c^{\frac{2}{3}}(a^{\frac{2}{3}}-c^{\frac{2}{3}})^2}\right\}-\frac{3ac}{2b^2}\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{\frac{2}{3}}}\right\}$$

$$\times\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}\Bigg]\int_0^b\left(b^{\frac{2}{3}}-y^{\frac{2}{3}}\right)^3dy.$$

$$\therefore\ V=\frac{4}{35}\pi b^3+\frac{8}{35}\ abc\ \sin^{-1}\left\{\frac{b^{\frac{2}{3}}-c^{\frac{2}{3}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}^{\frac{1}{3}}-\frac{8}{35}\ b^3\sin^{-1}\left\{\frac{a(b^{\frac{2}{3}}-c^{\frac{2}{3}})}{b\ (a^{\frac{2}{3}}-c^{\frac{2}{3}})}\right\}^{\frac{1}{3}}$$

$$-\frac{16}{35}\ abc\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{5}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}(c^{\frac{2}{3}}-a^{\frac{2}{3}})^2}\right\}$$

$$-\frac{8}{35}\ abc\left\{\frac{a^{\frac{2}{3}}c^{\frac{2}{3}}-b^{\frac{4}{3}}}{a^{\frac{2}{3}}c^{3}}\right\}\left\{\frac{(b^{\frac{2}{3}}-c^{\frac{2}{3}})^{\frac{1}{2}}(a^{\frac{2}{3}}-b^{\frac{2}{3}})^{\frac{1}{2}}}{a^{\frac{2}{3}}-c^{\frac{2}{3}}}\right\}.$$

Cor. If $b=c$, $V=\frac{4}{35}\pi b^3$.

If $a=b$, $V=\frac{4}{35}\pi a^2c$.

Also solved by *M. C. STEVENS.*

PROBLEMS.

18. Proposed by J. M. BANDY, Professor of Mathematics, Elon College, North Carolina.

If the ordinate ST of any point T on a circle

$$x^2+y^2=r^2$$

be produced so that $ST.TP=r^2$, prove that the whole area between the locus of P and its asymptotes is double the area of the circle.

19. Proposed by A. L. FOOTE, No. 830, Broad Street, New York City.

A and B are in a circular room 30 feet in diameter, A being at the center and B at the circumference. B runs around at the rate of 600 feet per minute and A persues him at the rate of 100 feet per minute. How long will the race last, and how far will each have traveled till B is caught.

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